Performance Analysis of the IEEE 802.11 MAC Protocols over a WLAN with Capture Effect

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ABSTRACT

In this paper, we use a discrete Markov chain model to analyze the throughput of CSMA/CA protocols in a wireless local area network (WLAN) considering the capture phenomenon, which means the packet with the strongest power may capture the receiver even in the presence of other overlapping packets. The model of capture effect over the theoretical throughput of CSMA/CA protocols in the presence of path loss, shadowing, and Rayleigh fading are obtained and compared with the model without capture effect.

Keywords: Performance analysis, ad hoc network, capture effect, propagation model.

1 Introduction

A wireless local area network (WLAN) provides an attractive networking alternative which enables a flexible mobility of wireless terminals and can avoid rewiring when the terminals are relocated. A WLAN employs the wireless channel for interconnecting terminals to form an ad hoc network or an infrastructure network with access points (APs). Since a WLAN offers the benefits of traditional wired LAN while greatly increasing flexibility, the WLAN based on the IEEE 802.11 specification has been widely deployed. In order to allow multiple wireless terminals to share a common channel, a medium access control (MAC) protocol is defined in the IEEE 802.11 specification [1]. One of the most important research issue in WLANs is the design and analysis of MAC protocols [2]-[4]. Today most existing research works on the performance analysis of MAC protocols assume that the wireless channel is noiseless and all packets arrive at the receiver with the same power level. Whenever two or more packets arrive at the receiver during the overlapping time period, they collide and all packets involved are destroyed. These models, reasonable in some communication environment, turn out to be too pessimistic in others. In a practical wireless network, the transmitted packets experience not only noise but also fading, so that the receiver may fail to detect the faded packets even though there is no collision. On the other hand, a packet can be received successfully in the presence of other overlapping packets if its power is larger than the interfering power by a certain margin. The later phenomenon is called capture effect. The capture effect can reduce the probability of collision and result in an increase of the system throughput. There are many studies for ALOHA protocols in a fading channel and with shadowing effect [5]-[7]. However, the capture effect on CSMA/CA protocols has not been well studied and reported except two recent publications [9] [10] in which fading, shadowing, and path loss effect are considered. In [9], the authors analyzed the influence of capture phenomenon over theoretical throughput and delay of a traffic-saturated IEEE 802.11b BSS (basic service set) and ad hoc configurations based on the model developed by Bianchi [3]. In [10], the authors assumed the CSMA/CA protocol as a hybrid protocol of slotted 1-persistent CSMA and p-persistent CSMA. The value of p is related to the backoff delay. However, it was left open on how to determine the p and the authors assumed some fixed values in their numerical results. In this paper, we derive a relatively realistic model of CSMA/CA protocol where fading, shadowing and near-far effect are considered. Furthermore, the traffic load in this paper is general which varies from light to heavy (saturated).

The rest of this paper is organized as follows. Section 2 gives the descriptions of system model, channel model, and capture model. The throughput of CSMA/CA protocols is analyzed in Section 3. In Section 4, some numerical results and discussions are provided. Finally, we conclude this paper in Section 5.

2 System Description

2.1 System Model

We consider a WLAN system with an AP surrounded by M wireless terminals. The wireless terminals are randomly distributed in a circle of unit radius around the AP. It is assumed that the locations of the wireless terminals are independent variables. The corresponding probability density function (pdf) of the distance between a wireless terminal and the AP is denoted as $f(r)$. Only the uplink communications in which wireless terminals attempt to send packets over a common shared channel to the AP are studied in this paper. The MAC protocols are implemented using CSMA/CA protocols. The time is slotted with a slot size α and the packets are allowed to transmit only at the beginning of a time slot. All packets are assumed to have the same length T_p. To simplify the analysis, we normalize the time by the packet length T_p. That is, the duration of a packet transmission equals to unit time of 1 and is composed of $1/\alpha$ time slots. The propagation delay between a wireless terminal and the AP is assumed to be the same and equals to a time slot. At the end of each time slot, every wireless terminal will be in either thinking state or backlogged state. In the thinking state, each terminal generates a new packet with probability g during a time slot. If the packet transmission is successful, the wireless terminal stays on the thinking state. A wireless terminal is said to be in the backlogged state if its transmission either had a channel collision or has been blocked because of a busy channel. No any new packet is generated in the backlogged state. A backlogged terminal remains in the backlogged state until it completes a successful transmission, at that time it switches to the thinking state. If we assume that the arrival process of packets G is Poisson process and g denotes a packet arrival rate during a time slot, we have $Mg = \alpha G$ [11]. Note that G (total traffic load) denotes the number of total packets in the system during one normalized unit time, while g denotes the packet arrival rate of one terminal during a time slot.

2.2 Channel Model

The channel model is characterized by three nearly independent, multiplicative propagation mechanisms, namely, path loss, shadowing, and fading. The path loss is proportional to $r^{-\beta}$, where β is the path loss exponent. The value of β depends on the propagation environment, typically taking values of 2 to 6, and is typically equal to 4 in land radio environment. The path loss effect gives rise to the near-far effect and determines the area mean power w_a. The area mean power is the received signal power in the absence of shadowing. The shadowing is described by a log-normal distribution of the local mean w_l about the area mean power w_a and is assumed to be superimposed on the path loss effect. If the local mean power is expressed in nepers, it has normal distribution about the area mean, with a logarithmic standard deviation σ_s. The multipath reception causes Rayleigh or Rician fading. Rayleigh fading causes the instantaneous received power to be exponentially distributed random variable. Taking Rayleigh fading, log-normal shadowing, and near-far effects into account, the uncondition probability density function of the instantaneous power w_s of a received packet at the AP is given by [5]

$$\begin{aligned} f_{w_s}(w_s) &= \int_0^\infty \int_0^\infty \frac{1}{w_l} \exp(-\frac{w_s}{w_l}) \frac{f(r)}{\sqrt{2\pi}\sigma_s w_l} \\ &\cdot \exp(-\frac{ln^2(r^\beta w_l)}{2\sigma_s^2}) dr dw_l, \end{aligned} \tag{1}$$

where $f(r)$ is the pdf of the distance describing the spatial distribution of the offered packet traffic around the AP. For example, we consider a uniform spatial distribution in which wireless terminals are uniformly distributed. In this case, the pdf is given by $f(r) = 2r$ and $r \in (0, 1)$.

2.3 Capture Model

The instantaneous powers received at the AP for different terminals will generally not be the same due to the different propagation decays. Therefore, even if there are more than two wireless terminals transmitting their packets at the same time, one of them may be successfully received at the AP. In comparing power levels of different packets, it is usually assumed that the power levels remain constant during the packet reception period, i.e., the power of each bit in a packet is the same. This assumption has been used earlier in [5]-[7], and is considered to be accurate if the users are stationary or are moving very slowly. If the wireless terminals move fast, the signal power may vary over the duration of a packet and all the packets received during a time slot should be compared on a bit-by-bit basis. Several capture models has been proposed in the literature [8]. In this paper, we use the model where a packet will be successfully received if the power of the concerned packet exceeds the joint power of the other interfering packets by at least a capture ratio z. That is,

$$w_0 > z\{\sum_{i=1}^{N} w_i + \eta\}, \tag{2}$$

where w_0 is the power of concerned packet, w_i ($i = 1, 2, ..., N$) is the power of interfering packet i, and η is the power of additive white Gaussian noise. It is of no consequence to ignore the noise since the CSMA/CA channel was principally contention limited, the same as ALOHA channel [5] [8]. Therefore, ignoring the effect of noise, the probability of capture can be expressed as

$$P_{cap}(N) = Pr\{w_0 > z \sum_{i=1}^{N} w_i\}. \tag{3}$$

In order to find the probability of capture, we need to know the joint pdf of the interfering packets. If the interference power is due to incoherent accumulation of N independently fading signals, the joint pdf is the N-fold convolution of the pdf of the individual signal power. Therefore, the probability of capture, given that N+1 wireless terminals transmit packets at the same time, can be obtained by

$$\begin{aligned} P_{cap}(N) &= \int_0^\infty \int_0^\infty \frac{f(r)}{\sqrt{2\pi}\sigma_s w_l} \exp(-\frac{ln^2(r^\beta w_l)}{2\sigma_s^2}) \\ &\cdot [\phi(\frac{z}{w_l})]^N dr dw_l, \end{aligned} \tag{4}$$

where $\phi(\cdot)$ is the Laplace image of the pdf of one single interferer. Using (1), $\phi(\cdot)$ can be expressed as

$$\phi(s) = \int_0^\infty \int_0^\infty \frac{1}{1 + sw_l} \frac{f(r)}{\sqrt{2\pi}\sigma_s w_l} \exp(-\frac{ln^2(r^\beta w_l)}{2\sigma_s^2}) dr dw_l. \tag{5}$$

The probability that one out of $N + 1$ packets captures the AP is given by

$$q_N = (N + 1) P_{cap}(N). \tag{6}$$

3 Throughput for CSMA/CA Protocols

In this section, we use the approach described in our previous research work [2] to evaluate the throughput for a finite number of wireless terminals in a slotted CSMA/CA system with the models described in Section 2. We assume that the channel state consists of a sequence of regeneration cycles composed of idle period $\overline{I}$ and busy period $\overline{B}$. Let $\overline{A}$ be the average time spent in useful transmission during a regeneration cycle. The throughput S is defined as the fraction of channel time occupied by a valid transmission and can be obtained by

$$S = \frac{\overline{A}}{(\overline{B} + \overline{I})}. \tag{7}$$

Let $X(t)$ be the number of wireless terminals in the backlogged state. The random process $\{x(t) = i\}$ can be modeled by a homogeneous Markov chain identified by the last slot of each idle period (see Figure 1). Since there are M wireless terminals in the system, $X(t)$ can be 0, 1, 2,..., M. Thus, the embedded Markov chain for $X(t)$ has $M + 1$ states as shown in Figure 2. The transition from state i to state j $(i \leq j)$ means that there are some thinking terminals entering to the backlogged state. Similarly, the transition from state $i + 1$ to state i represents that there is a successful packet transmission. It is assumed that each backlogged terminal has the same steady-state probability ν_i to send a packet at the time slot t when $X(t)$ equals to i. It is also assumed that the acknowledgments from the AP to wireless terminals are received perfectly. In order to determine the probability ν_i, we need to know the collision probability. In [4], an analytical model is developed to compute the collision probability $P_c(i)$, which is given as

$$P_c(i) = 1 - \left[1 - \frac{2\,(1 - 2P_c(i))}{1 - P_c(i) - 2^m P_c^{m+1}(i)} \frac{1}{W}\right]^{i-1}, i > 1, \tag{8}$$

where W is the minimum contention window and m is to determine the maximum contention window W_{max} and it satisfies $W_{max} = 2^m W$. Note that the collision probability $P_c(i)$ is the probability that more than one backlogged terminals transmit at the same time slot. This yields to

$$P_c(i) = 1 - (1 - \nu_i)^{i-1}. \tag{9}$$

Then we can get the probability ν_i $(i > 1)$ from equations (8) and (9). Obviously, $\nu_0 = 0$ and $\nu_1 = 1/W$.

Our goal is to obtain the stationary distribution of the Markov chain

$$\pi_i = \lim_{t \to \infty} P\{x(t) = i\}. \tag{10}$$

For that purpose, we need to find the transition probability matrix **P**. Using the linear feedback model introduced by Tobagi and Kleinrock [12] [13], **P** is the product of several single time slot transition matrices which we will define next. We denote the transition matrix by **R** for time slot $t_1 + I$ and **Q** for all remaining time slots of the busy period. Since the length of the busy period depends on the number of terminals which become ready in time slot $t_1 + I$, we have $\mathbf{R} = \mathbf{U} + \mathbf{F}$, where the (i, k)th elements of **U** and **F** are defined as

$$u_{ik} = P\{X(t_1 + I + 1) = k \text{ and transmission is successful} | X(t_1 + I) = i\} \tag{11}$$

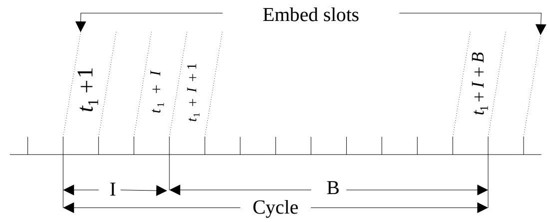

Figure 1: Embedded slots

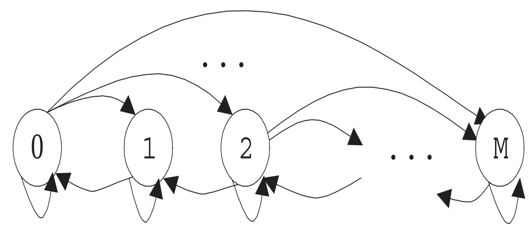

Figure 2: Markov chain model for the entire network

and

$$f_{ik} = P\{X(t_1 + I + 1) = k \text{ and transmission is unsuccessful} | X(t_1 + I) = i\} \tag{12}$$

Note that **U** means that there is only one node ready to transmit or the packet captures the receiver if more than one packets are ready to transmit in time slot $t_1 + I$. **F** means that there are more than one terminals ready to transmit in time slot $t_1 + I$ and the packets do not capture the receiver. **Q** reflects the addition to the backlogged states from the $M - X(t)$ thinking terminals in any time slot t during the busy period. As we can see, if the transmission is successful, the busy period has length T; if it is unsuccessful, the busy period is C.

According to [12] [13], the transmission matrix **P** is expressed as

$$\mathbf{P} = \mathbf{U}\mathbf{Q}^T\mathbf{J} + \mathbf{F}\mathbf{Q}^C, \tag{13}$$

where J represents the fact that a successful transmission decreases the number of backlogged terminals by 1 and the elements of matrices **U**, **F**, **Q**, and **J** are given by

$$q_{ik} = \begin{cases} 0, & k < i, \\ \begin{pmatrix} M - i \\ k - i \end{pmatrix} (1 - g)^{M-k} g^{k-i}, & k \geq i + 1, \end{cases} \tag{14}$$

$$j_{ik} = \begin{cases} 1, & k = i - 1, \\ 0, & otherwise. \end{cases} \tag{15}$$

$$u_{ik} = \begin{cases} 0, & k < i, \\ \frac{(1-g)^{M-i}\left[i\nu_i(1-\nu_i)^{i-1}+\sum_{j=2}^{i}\binom{i}{j}\nu_i^j(1-\nu_i)^{i-j}q_i\right]}{1-(1-\nu_i)^i(1-g)^{M-i}}, & k = i, \\ \frac{(M-i)g(1-g)^{M-i-1}\left[(1-\nu_i)^i+\sum_{j=1}^{i}\binom{i}{j}\nu_i^j(1-\nu_i)^{i-j}q_{i+1}\right]}{1-(1-\nu_i)^i(1-g)^{M-i}}, & k = i+1, \\ \frac{\binom{M-i}{k-i}(1-g)^{M-k}g^{k-i}\left[\sum_{j=0}^{i}\binom{i}{j}\nu_i^j(1-\nu_i)^{i-j}q_{k+j-i}\right]}{1-(1-\nu_i)^i(1-g)^{M-i}}, & k > i+1. \end{cases} \tag{12}$$

$$f_{ik} = \begin{cases} 0, & k < i, \\ \frac{(1-g)^{M-i}\left[\sum_{j=2}^{i}\binom{i}{j}\nu_i^j(1-\nu_i)^{i-j}(1-q_i)\right]}{1-(1-\nu_i)^i(1-g)^{M-i}} 1-(1-\nu_i)^i(1-g)^{M-i}, & k = i, \\ \frac{(M-i)g(1-g)^{M-i-1}\left[\sum_{j=1}^{i}\binom{i}{j}\nu_i^j(1-\nu_i)^{i-j}(1-q_{i+1})\right]}{1-(1-\nu_i)^i(1-g)^{M-i}}, & k = i+1, \\ \frac{\binom{M-i}{k-i}(1-g)^{M-k}g^{k-i}\left[\sum_{j=0}^{i}\binom{i}{j}\nu_i^j(1-\nu_i)^{i-j}(1-q_{k+j-i})\right]}{1-(1-\nu_i)^i(1-g)^{M-i}}, & k > i+1. \end{cases} \tag{13}$$

The steady-state probabilities of the Markov process are defined as a row vector $\pi = [\pi_0, \pi_1, ...\pi_N]$, which can be determined by $\pi = \pi\mathbf{P}$.

Since the idle period is geometrically distributed [11], its expectation is given by

$$\overline{I_i} = \frac{1}{1-(1-\nu_i)^i(1-g)^{M-i}}. \tag{16}$$

The probability of successful transmission $P_s(i)$ when $X(t)$ equals to i is given by

$$P_s(i) = \sum_{k=0}^{M} u_{ik}. \tag{17}$$

Therefore, the throughput can be obtained by

$$S = \frac{\sum_{i=0}^{M}\pi_i P_s(i)T_p}{\sum_{i=0}^{M}\pi_i\{\overline{I_i} + P_s(i)T + [1-P_s(i)]C\}}. \tag{18}$$

The values of T and C differ depending on the access model. For basic CSMA/CA protocol:

$$\begin{cases} T = DIFS + T_p + SIFS + ACK + 2\alpha \\ C = DIFS + T_p + SIFS + \alpha \end{cases}$$

and CSMA/CA with RTS/CTS protocol:

$$\begin{cases} T = DIFS + RTS + CTS + T_p + 3SIFS + ACK + 4\alpha \\ C = DIFS + RTS + \alpha \end{cases}$$

4 Numerical Results and Discussions

Numerical values are computed in this section based on the analysis presented in the previous sections. In all the plots, the following parameters are assumed: (a) the path loss exponent $\beta = 4$; (b) the wireless terminals are uniformly distributed in a circular area with radius 1.

The capture probability can be obtained from equation (6) using the Gauss-Hermite quadrature [14]. Figure 3 plots the capture probability versus the number of contending terminals, for some values of capture ratio z. The behavior of Figure 3 shows that the capture probability decreases as the number of contending terminals increases. One can observe that an increase of successful capture probability occurs when z decreases. Lower value of z means more powerful receiver detection probability. On the other hand, the increase of transmission power should be limited due to the power control and battery requirement.

Figure 4 shows the throughput versus total offered load G given the capture ratio z equals to 10. The other parameters are defined as follows: $M = 15$, $T_p = 1$ (i.e.,100 slots), $\alpha = 0.01$, $DIFS = 0.03$, $SIFS = 0.01$, $RTS = 0.05$, $CTS = 0.05$, $ACK = 0.05$, $W = 32$, and $W_{max} = 1024$. If the basic CSMA/CA is used, it is obvious that the presence of capture effect generates significant throughput improvement as the total traffic load G increases. For example, given $G = 3.0$, the throughput with capture effect is estimated to 0.82 as opposed to 0.73 in the absence of cap-

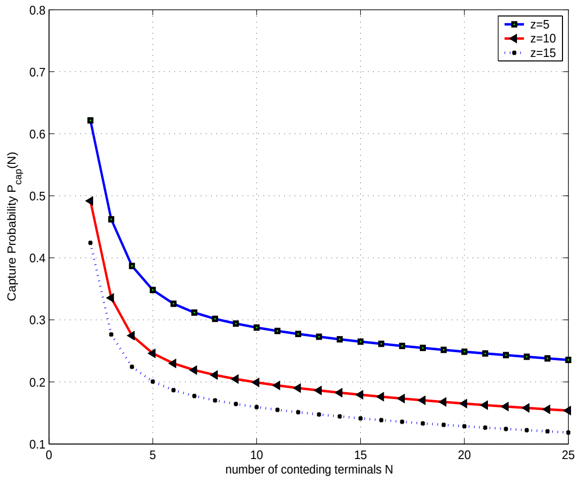

Figure 3: Capture probability versus the number of contending terminals ($\sigma_s = 1.35$)

ture effect. However, for CSMA/CA with RTS/CTS, the exchange of RTS and CTS before the actual transmission significantly reduces the likeliness of simultaneous transmission and packet capture. Moreover, we note that the throughputs of CSMA/CA protocols in the fading channel are not much different compared with those in the perfect channel model when the traffic load is low. This is because we ignore the noise and assume that the transmission is always successful if only one terminal transmits packet.

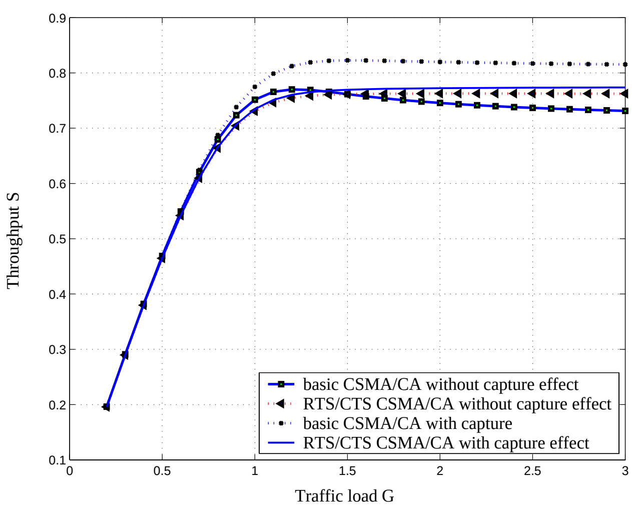

Figure 4: Throughput of CSMA/CA protocols versus traffic load G

5 Conclusions

The CSMA/CA system in the mobile radio networks was investigated in the presence of path loss, shadowing, and fading. It is shown that path loss, shadowing, and fading make the capture effect possible and provide the CSMA/CA system with substantial improvement in the throughput.

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